

Simulation, design and thermal analysis of a solar Stirling engine using MATLAB



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ABSTRACT

This paper presents the modeling and simulation for a prototype of the solar-powered Stirling engine working at the low temperature range. A mathematical model for the thermal analysis of the solar-powered low temperature Stirling engine with heat transfer is developed using Matlab program. The model takes into consideration the effect of the absorber temperature on the thermal analysis like as radiation and convection heat transfer between the absorber and the working fluid as well as radiation and convection heat transfer between the lower temperature plate and the working fluid. Hence, the present analysis provides a theoretical guidance for designing and operating of the solar-powered low temperature Stirling engine system, as well as estimating output power from the solar Stirling engine using Matlab program. This study attempts to demonstrate the potential of the low temperature Stirling engine as an option for the prime movers for Photovoltaic tracking systems. The heat source temperature is 40–60 °C as the temperature available from the sun directly.

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1. Introduction

Solar energy is the energy of sunlight collected and used to provide electricity, heating, cooling and other purposes. This clean energy reduces the consumption of fossil fuels and also reduces the environmental pollutants [1]. Solar energy is one of the more attractive renewable energy sources that can be used as an input energy source for heat engines. In fact, any heat energy source can be used with the Stirling engine [2].

The direct conversion of solar power into mechanical power reduces both the cost and complexity of the prime mover. Stirling engines are able to use solar energy that is a cheap source of energy. The increasing interest in Stirling engines is largely due to the fact the engine is more environmentally friendly than the widely used internal combustion engine [3]. Stirling engines are thermodynamic devices working theoretically on the Stirling cycle and using a compressible fluid, such as air, hydrogen, helium, nitrogen or even vapors, are used as working fluids. The Stirling engine offers possibility for having high efficiency engine with less exhaust emissions in comparison with the internal combustion engine. The earlier Stirling engines were huge and inefficient. In recent years, a number of new Stirling engine models have been developed to improve the deficiencies [2].

Tlili et al. [4] presented the engine works with a closed cycle and used several gases as working fluid. Glushenkov et al. [5] evaluated thermodynamic analysis of an unconventional heat engine. The engine studied has a number of advantages compared to state of the art Stirling engines. Cheng et al. [6] estimated theoretical analysis of the effects of the geometrical parameters on the shaft work of the Stirling engines. Campos et al. [7] showed a mathematical model to simulate the operation of Stirling engines. A thermodynamic optimization of the cycle for maximum thermal efficiency was conducted. Iskander Tlili et al. [8] estimated maximum power and efficiency at the maximum power point of an end reversible Stirling heat engine with finite heat capacitance rate of external fluids in the heat source/sink reservoirs with regenerative losses are treated. Cheng et al. [9] presented development of a dynamic model for the beta-type Stirling engines with rhombic-drive mechanism. Cheng et al. [10] presented numerical predictions of relationship between operating speed and shaft power output of Stirling engines.

Aksoy and Cinar [11] investigated the effect of the grooved displacer cylinder on engine performance from kinematic and thermodynamic points of view. Chen et al. [12] studied experimentally a helium charge γ -type twin power piston Stirling engine to understand the effects of several regenerator parameters on the overall performance of the engine. Formosa and Despesse [13] presented an analytical model for design and optimization of FPSE. To deal with the discrepancy between the high theoretical efficiency of Stirling and a constructed prototypes the major losses are be taken

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Nomenclature

SL	standard longitude, min	$F_{a_1-p_1}$	the view factor coefficient between point a1 & Plate 1
LL	local longitude, min	$F_{p_2-a_2}$	the view factor coefficient between Plate 2 & point a2
L	local latitude, min	A_{p_1}	the Area of the Plate 1, m ²
δ	the declination angle, °	A_{p_2}	the Area of the Plate 2, m ²
h	the hour angle, °	σ	the Stefan–Boltzmann constant ($5.669 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$), W/m ² K ⁴
Φ	the Zenith angle, °	ε_{p_1}	the Plate 1 emissivity
θ	the solar incidence angle, °	ε_{p_2}	the Plate 2 emissivity
β	the surface tilt angle from the horizontal, °	h_c	the connective heat transfer coefficient, W/m ² K
G_t	the global irradiance, W/m ²	K	the thermal Conductivity of Fluid (air), W/m K
G_{B_t}	total beam solar radiation on a tilted surface, W/m ²	L	the length of fluid layer = 1.5 cm = 0.015 m, m
G_{D_t}	total diffuse solar radiation on a tilted surface, W/m ²	N_u	the Nusselt Number
G_{R_t}	total ground-reflected solar radiation on a tilted surface, W/m ²	G_r	the Grasshof number
G_B	beam radiation on a horizontal surface, W/m ²	P_r	the Prandtl number
G_{B_n}	beam radiation in the direction of the rays, W/m ²	a, n	the constants based on fluid circulation
R_B	the beam radiation tilt factor	β_T	the coefficient of thermal expansion, 1/K
R_D	the diffuse radiation tilt factor	μ	the dynamic viscosity of Fluid (air), kg/m s
R_R	the ground-reflected radiation tilt factor	ρ	the density of Fluid (air), kg/m ³
ρ	the ground reflectance, %	g	the gravity of acceleration = 9.8 m/s ² , m/s ²
T_{air}	the temperature of the container air of Stirling engine, K	(T_1-T_2)	the temperature difference, K
$T_{p_1-p_2}$	the temperature of upper and lower layer of Stirling engine, K	c_p	the specific heat, kJ/kg K
T_{a_1}	the temperature of the surrounding air in sun area above Plate 1, K	r_o, r_i	outer radius & inner radius of Plate 1, m
T_{a_2}	the temperature of the surrounding air below Plate 2, K	L	length of air fluid inside heating flow, m
$d_{p_1-p_2, air}$	the thickness of each layer, m	D_1, D_2	diameter of Plate 1 & Plate 2, m
$\rho_{p_1-p_2, air}$	the density of each layer, kg/m ³	K_{air}	the conductivity of air, W/m K
$C_{p_{p_1-p_2, air}}$	the heat capacity of each layer, J/kg K	μ_{air}	the viscosity of air, kg/m K
$q_{p_1-p_2}$	the amount of solar radiation falling on the surface of each layer, W/m ²	P	the power output of the Stirling engine, watts
G_t	the total solar radiation on a surface, W/m ²	P_m	the mean average gas pressure in the container of Stirling engine, (Pa) or (MPa, if volume is in cm ³)
α_{p_1}	the absorptance of Plate (1) = 0.95 (Plate Black Paint)	V	the swept volume of the expansion space in the container of Stirling engine, (m ³) or (cm ³ , if pressure is in MPa)
α_{p_2}	the absorptance of Plate (2) = 0.25 (Plate Silver Paint)	F	the engine cycle frequency of Stirling engine container, Hz
$h_{c, a_1-p_1}, h_{c, 1rp_2-a_2}$	the convection heat transfer coefficients outside Stirling, W/m ² K	B_n	the Beale number depend on air heater temperature of Stirling engine
$h_{c, 1rp_1-air}, h_{c, air-p_2}$	the convection heat transfer coefficients inside Stirling, W/m ² K	ρ_{air}	the density of air (=1.127 kg/m ³), kg/m ³
$F_{p_1-p_2}$	the view factor coefficient between Plate 1 & Plate 2	R	the constant of proportionality, is known as the universal gas constant (=287 J/kg K), J/kg K
		T_{air}	the temperature of the container air of Stirling engine, K

into account. Ahmadi et al. [14] presented a thermodynamic model to determine output power and thermal efficiency of the solar-dish Stirling heat engine. A final optimal solution was selected from available solutions of the Pareto frontier using three decision making methods including LINMAP, TOPSIS and fuzzy methods. Then, Ahmadi et al. [15] presented finite-time thermodynamics to determine output power, thermal efficiency and rate of entropy generation of the Solar Dish-Stirling engine and factors such as dish collector performance, conductive and radiative heat transfer mechanism at heat source and heat sink of the engine are involved in the analysis.

Sripakagorn et al. [16] developed a prototype Stirling engine working at the moderate temperature range. Abdullah et al. [17] presented the design considerations to be taken in designing of a low temperature differential double-acting Stirling engine for solar application. Li et al. [18] developed a mathematical model for the

overall thermal efficiency of the solar-powered high temperature differential dish-Stirling engine with finite-rate heat transfer, regenerative heat losses, conductive thermal bridging losses and finite regeneration processes time. Kongtragool et al. [19] provide a theoretical investigation on the thermodynamic analysis of a Stirling engine. Kongtragool et al. [2] provided a literature review on solar-powered Stirling engines and low temperature differential Stirling engines technology.

2. Design of solar-powered low temperature Stirling engine

In this paper, a low temperature Stirling engine has been developed. This low temperature Solar-powered Stirling motor design was conceived for the transduction of solar energy at a necessary low temperature difference of 10:20 °C as shown in Fig. 1. During

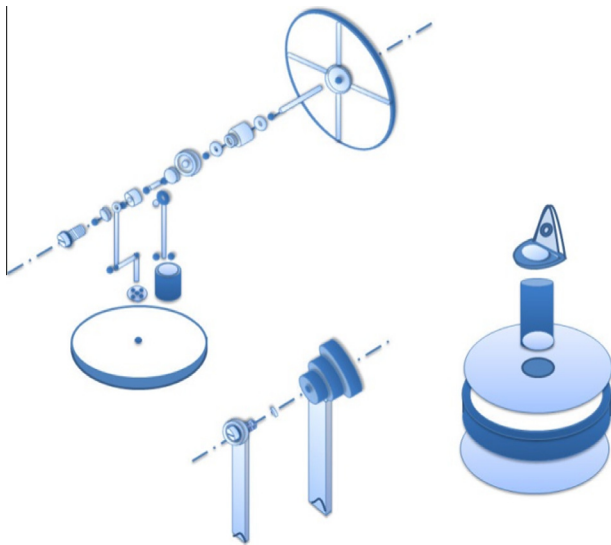


Fig. 1. Stirling engine 3-D design.

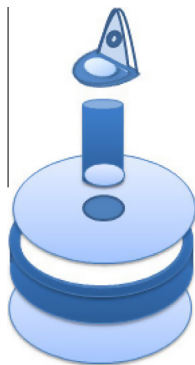


Fig. 2a. Stirling engine (Heating Cylinder) design.

the construction of the low temperature solar-powered Stirling engine, an emphasis was put on temperature differences between two plates upper and lower, which is the important condition for operating the engine.

This configuration uses separated cylinders for the displacer and the power pistons, with the power cylinder connected to the displacer cylinder. The displacer moves working fluid between the hot space and the cold space of the displacer cylinder (Heating Cylinder). In this configuration, the power piston both compresses and expands the working fluid [2,20].

A solar-powered Stirling engine has a single power piston (26) as shown in Figs. 3a–3c arranged within the power cylinder (2) in Figs. 2a–2c on the same shaft as a displacer piston (27) in Figs. 3a–3c. The displacer piston is a loose fit and does not extract any power from the expanding gas but only serves to shuttle the working gas between the hot and cold heat exchangers. When the working gas is pushed to the hot end of the cylinder it expands and pushes the power piston. When it is pushed to the cold end of the cylinder it contracts and the momentum of the machine, usually enhanced by a flywheel, pushes the power piston the other way to compress the gas. Unlike the alpha type, the beta type avoids the technical problems of hot moving seals.

In this way, the movement of the working piston, discharger and crank mechanism can be visualized. The sealing of the heat transduction space is ensured by two O-rings, which simultaneously serve to secure the upper and lower plates with the

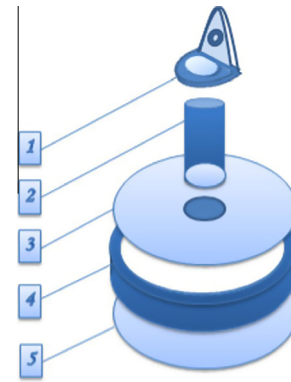


Fig. 2b. Stirling engine (Heating Cylinder components) design. 1. Elbow Joint, 2. Power Cylinder, 3. Top Plate, 4. Heating Cylinder, 5. Lower Plate.

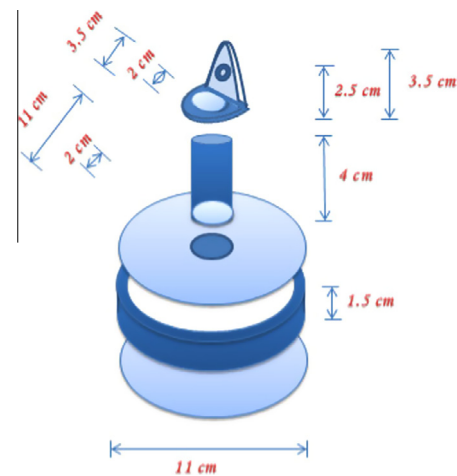


Fig. 2c. Stirling engine (Heating Cylinder dimensions) design.

casing wall. Assembly is achieved by a re-sealable Snap-on connection.

To minimize friction, the crankshaft (13) as shown in Figs. 3a–3c and both connection rods (3, 4) as shown in Figs. 4a and 4b are mounted in precision miniature ball bearings (23). Since the working piston runs in a dry cylinder, the low temperature solar-powered Stirling engine is extremely maintenance resistant.

The upper and lower Plates (3 and 5) were made from 0.3 cm thick aluminum plate. The plates were then machined on a Bridgeport mill, using a rotary table. To maintain concentricity, the upper and lower plates were machined simultaneously as shown in Figs. 2a–2c.

The displacer (27) is a special-purpose piston, used in this type of solar Stirling engine, to move the working gas back and forth between the hot and cold heat plates. Depending on the type of engine design, the displacer is sealed to the cylinder; it may be a loose fit within the cylinder, allowing the working gas to pass around it as it moves to occupy the part of the cylinder inside. The displacer (27) itself was made from 0.6 cm thick high density foam board, much like the blue foam used for insulation in the construction of houses. The foam disk was easily formed into place with the help of a belt sander. Proper thickness of 0.8 cm was obtained by a band saw, and cleaned off by the belt sander as shown in Figs. 3a–3c.

The displacer rod (21) was supposed to be made out of steel, but due to limited resources the only 0.127 cm rod that was available was brass-welding rod as shown in Figs. 3a–3c.

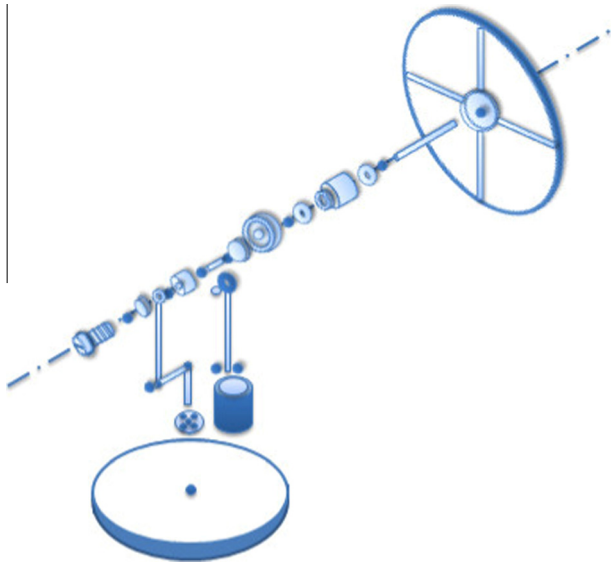


Fig. 3a. Stirling engine (rotating part) design.

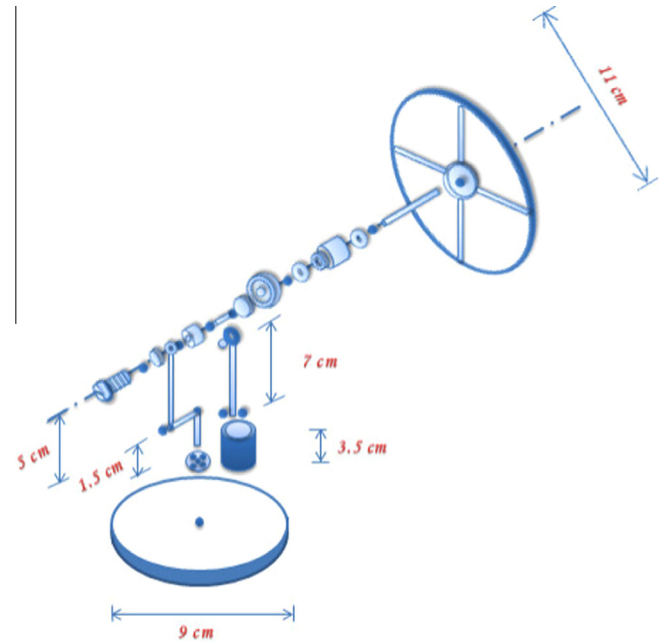


Fig. 3c. Stirling engine (rotating part dimensions) design.

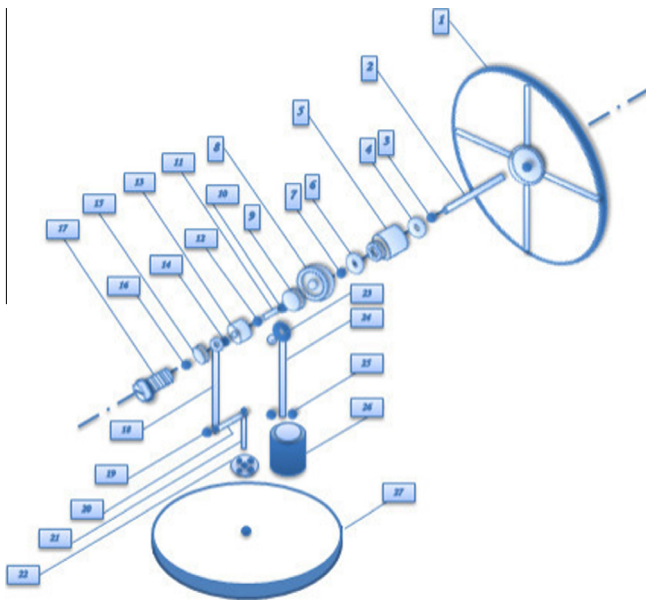


Fig. 3b. Stirling engine (rotating part components) design. 1. Flywheel, 2. Crankshaft, 3. Shim, 4. Ball Bearing, 5. Bearing Bush, 6. Ball Bearing, 7. Shim, 8. Crank Disc, 9. Socket, 10. Shim, 11. Crank Pin, 12. Shim, 13. Crank Disc, 14. Shim, 15. Ball Bearing, 16. Shim, 17. Head Screw, 18. Connecting Rod, Short, 19. Adjusting Collar, 20. Connection Cylinder, 21. Displacer Rod, 22. Displacer Bushing, 23. Ball Bearing, 24. Connecting Rod, Long, 25. Cylinder Pin, 26. Power Piston, 27. Displacer.

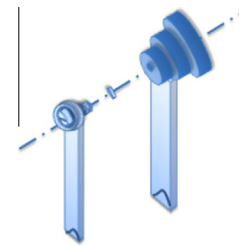


Fig. 4a. Stirling engine (connection part) design.

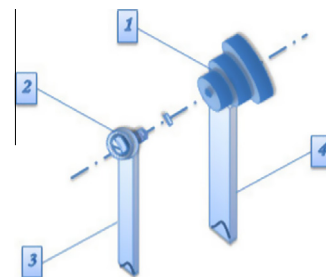


Fig. 4b. Stirling engine (connection part components) design. 1. Connection Ball Bearing and Rings, 2. Head Screw, 3. Connecting Rod, Short, 4. Connecting Rod, Long.

The bearing plate (5) was cut out of the same piece of aluminum that the top and bottom plates came from. The bearing housing (4, 6) and collar (19) were made from aluminum using a lathe. Other parts such as the crankpin (11), flywheel hub (1), crankshaft (8, 13), and displacer crankpin (15), were also turned on a lathe as shown in Figs. 3a–3c.

One of the most difficult pieces to machine was the Displacer Chamber Ring. It was a rough-cut Plexiglas ring with an inside diameter of 10.5 cm. The height of the ring had to be between 1.4 cm and 1.5 cm. An improper height would offset the volume inside the chamber. To aid in the making this piece, a template was made out of plywood. Using a Rotozip tool with a circular attachment, a path 0.5 cm was cut. The ring was then placed into the

groove and sanded smooth on a belt sander. As the plastic was sanded, the excess material seemed to melt away. The ring had to be perfectly flat on the top and bottom faces, to keep air from leaking around the ring when it was sandwiched between the upper and lower plates as shown in Figs. 2a–2c.

The flywheel (1) was made by adhering three compact disks together with epoxy. The rods connecting the piston (26) to the right bearing assembly (23), and the displacer rod (21) to the left bearing assembly. This material is rigid enough that it will not deform under normal usage, but easy enough to shape into the proper form using simple tools as shown in Figs. 3a–3c and 4a–b.

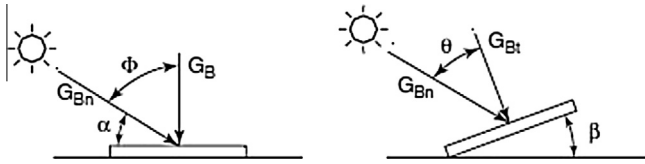


Fig. 5. Beam radiations on horizontal and tilted surfaces.

Table 1
Coefficients for 1990 Perez diffuse radiation model [23].

ε	f_{11}	f_{12}	f_{13}	f_{21}	f_{22}	f_{23}
1–1.1065	–0.008	0.588	–0.062	–0.060	0.072	–0.022
1.065–1.230	0.130	0.683	–0.151	–0.019	0.066	–0.029
1.230–1.500	0.330	0.487	–0.221	0.055	–0.064	–0.026
1.500–1.950	0.568	0.187	–0.295	0.109	–0.152	0.014
1.950–2.800	0.873	–0.392	–0.362	0.226	–0.462	0.001
2.800–4.500	1.132	–1.237	–0.412	0.288	–0.823	0.056
4.500–6.200	1.060	–1.600	–0.359	0.264	–1.127	0.131
>6.200	0.678	–0.327	–0.250	0.156	–1.377	0.251

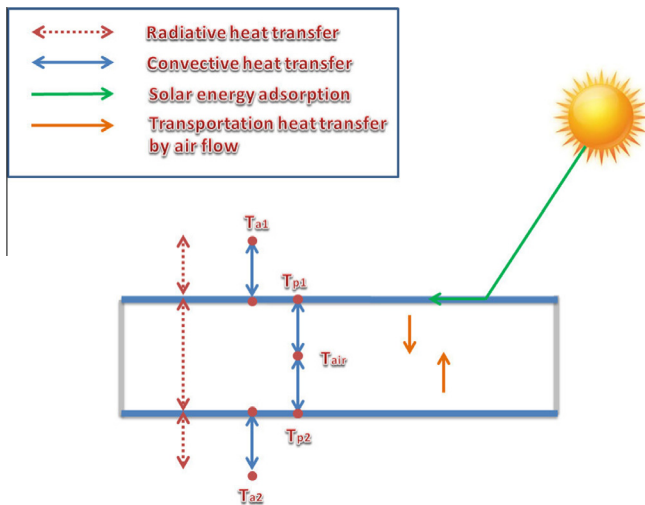


Fig. 6. Simplified system diagram with Stirling engine heating container and temperature in sides.

There are two positions in move Stirling engine:

- (1) When the displacer is near the bottom of the Heating Cylinder; inside the engine most of the gas has been heated by the heat source (sun energy) and it expands. This causes the pressure to increase which forces the piston down.
- (2) When the displacer is near the top of the Heating Cylinder; most of the gas in the engine has now cooled and contracts causing the pressure to decrease, which in turn allows the piston to move up and compress the gas.

3. Solar radiation modeling on tilted surfaces calculations

In this section a solar simulation has been made to estimate average global radiation using hours of bright sunshine and calculated solar radiation data available for solar powered Stirling engine in Egypt. In this part, an empirical model is proposed for estimating daily global solar radiation on a horizontal and on a tilted, south-facing surface by the day of the year. Also, presents the implementation of a generalized solar radiation simulation model using MATLAB/GUI interface. Global solar radiation is the

Table 2
Estimation of a and n constants.

Surface	(Gr-Pr)	a	n
Vertical plates/cylinders	10^4 to 10^9	0.59	0.25
	10^9 to 10^{12}	0.13	0.33
Horizontal pipes	10^3 to 10^9	0.53	0.25
Horizontal plates	10^5 to 2×10^7	0.54	0.25
Heated face up or cooled face down	2×10^7 to 3×10^{10}	0.14	0.33
Vertical plates heated face up or cooled face down	3×10^5 to 3×10^{10}	0.27	0.25

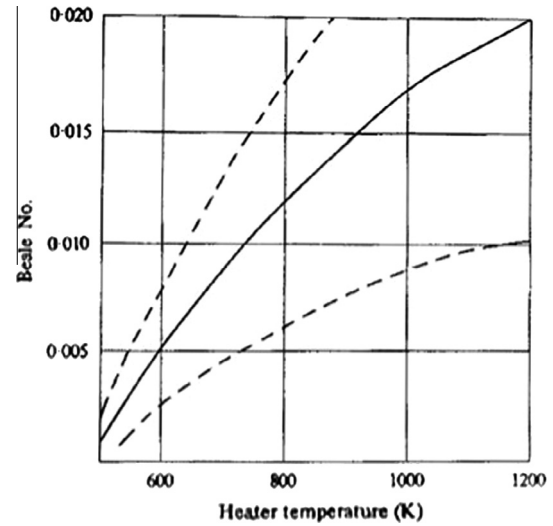


Fig. 7. Beale number as a function of source temperature [26].

algebraic sum of three solar components; first is the direct (beam) solar radiation, the second is the diffuse solar radiation, and the third is the ground reflected solar radiation [21]. A flat surface absorbs beam (G_{Bt}), diffuse (G_{Dt}) and ground-reflected (G_{Rt}) solar radiation; that is [22],

$$G_t = G_{Bt} + G_{Dt} + G_{Rt} \quad (1)$$

3.1. Beam radiation

The beam component is calculated using the direct normal irradiance (G_{Bn}) from the simulation model.

As shown in Fig. 5, the beam radiation on a tilted surface is

$$G_{Bt} = G_{Bn} \cos(\theta) \quad (2)$$

and on a horizontal surface,

$$G_B = G_{Bn} \cos(\Phi) \quad (3)$$

It follows that

$$R_B = \frac{G_{Bt}}{G_B} = \frac{\cos(\theta)}{\cos(\Phi)} \quad (4)$$

The beam radiation component for any surface is

$$G_{Bt} = G_B R_B \quad (5)$$

In Eq. (4), the zenith angle (Φ) and the incident angle (θ) can be Simplified. Therefore, Eq. (4) becomes

$$R_B = \frac{\cos(\theta)}{\cos(\Phi)} = \frac{\sin(L - \beta) \sin(\delta) + \cos(L - \beta) \cos(\delta) \cos(h)}{\sin(L) \sin(\delta) + \cos(L) \cos(\delta) \cos(h)} \quad (6)$$

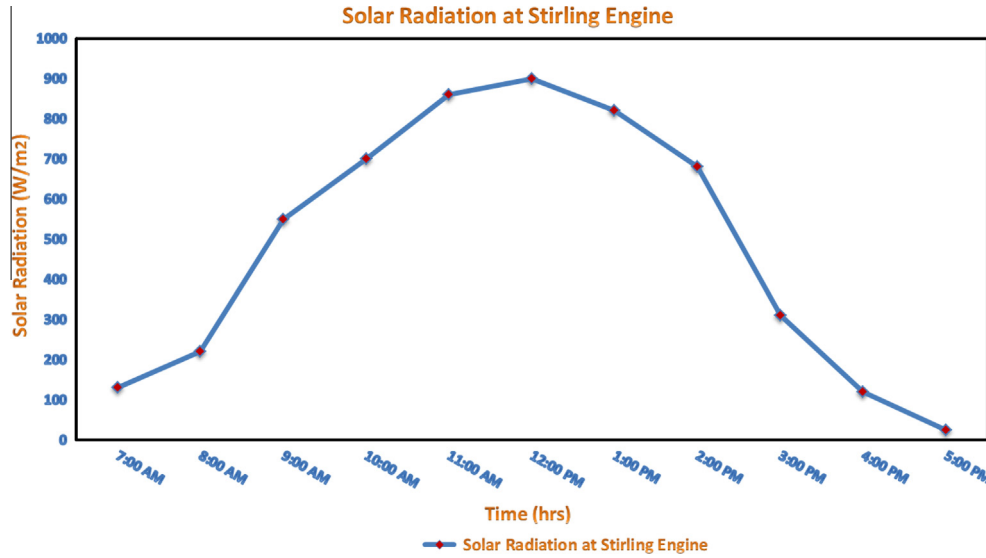


Fig. 8. Practical solar radiation data in Stirling engine heating container during day hours on 2 March 2013 in Giza.

Table 3

Data needed for solar radiation calculations in Giza city for Matlab simulation model [27].

Location	Egypt – Giza National Research Centre
The standard meridian, Egypt	31°17' East
The local longitude, Giza	31°10' East
The local latitude, Giza	30°00' North
Date	Changes over the year
Time	Changes over the day
The ground reflectance	0.27
Tilt angle, β	30

The ratio of beam radiation on the surface to that on a horizontal surface at any time is given by

$$R_B = \frac{\sqrt{(1 - \cos^2(\delta) \sin^2(h))}}{\sin(L) \sin(\delta) + \cos(L) \cos(\delta) \cos(h)} \quad (7)$$

3.2. Diffuse radiation

The diffuse radiation on a tilted surface is:

$$G_{Dt} = G_D R_D \quad (8)$$

$$G_{Dt} = (0.11 \times G_{Bn}) R_D \quad (9)$$

Three reflected radiation models were implemented previously: the first generation model of Liu and Jordan, the second generation irradiance model of Klucher, and the third generation 1990 model of Perez et al. (see Table 1 [23]). The three models differ in the computation of R_D . This Matlab software using first model (Isotropic Model).

In this model, the calculations of R_D is simple and given by

$$R_D = \frac{1 + \cos(\beta)}{2} \quad (10)$$

The diffuse radiation component of the Liu and Jordan [11,20] isotropic model G_{Dt} is

$$G_{Dt} = G_D \left(\frac{1 + \cos(\beta)}{2} \right) \quad (11)$$

3.3. Ground reflected radiation

The ground reflected radiation on a tilted surface is:

$$G_{Rt} = (G_B + G_D) R_R \quad (12)$$

The solar radiation simulation model and results will be presented in Section 6.

4. Thermal analysis modeling for solar-powered Stirling engine

The Stirling hot air engine is a simple type of engine that uses a compressible fluid as the working fluid. Because the working fluid is in a closed system, there are no problems with contamination and working fluid costs.

This part of paper presents the modeling and simulation for a prototype of the solar-powered Stirling engine working at the low temperature range. A mathematical model for the thermal analysis of the solar-powered low temperature Stirling engine with heat transfer is developed using Matlab program. The model takes into consideration the effect of the absorber temperature on the thermal analysis like as radiation and convection heat transfer between the absorber and the working fluid as well as radiation and convection heat transfer between the lower temperature plate and the working fluid as shown in Fig. 6. Hence, the present analysis provides a theoretical guidance for designing and operating of the solar-powered low temperature Stirling engine system, as well as estimating output power from the solar Stirling engine using Matlab Program as shown in Fig. 19. This program attempts to demonstrate the potential of the low temperature Stirling engine as an option for the prime movers for Photovoltaic tracking systems. The heat source temperature is 40–60 °C as the temperature available from the sun directly. The details of these results are presented in next section.

Eqs. (13)–(15) are a general relationship for energy balance equations for three points at plate 1 and plate 2 and at the container midpoint [24].

$$\alpha_{p_1} G_t + h_{c,a_1-p_1} (T_{a_1} - T_{p_1}) + \sigma \epsilon_{p_1} F_{a_1-p_1} (T_{a_1}^4 - T_{p_1}^4) = h_{c,1rp_1-air} (T_{p_1} - T_{air}) + \sigma \epsilon_{p_1} F_{p_1-p_2} (T_{p_1}^4 - T_{p_2}^4) \quad (13)$$

$$h_{c,air-p_2} (T_{air} - T_{p_2}) + \sigma \epsilon_{p_1} F_{p_1-p_2} (A_{p_1}/A_{p_2}) (T_{p_1}^4 - T_{p_2}^4) = h_{c,1rp_2-a_2} (T_{p_2} - T_{a_2}) + \sigma \epsilon_{p_2} F_{p_2-a_2} (T_{p_2}^4 - T_{a_2}^4) \quad (14)$$

$$h_{c,p_1-air} (T_{p_1} - T_{air}) = h_{c,air-p_2} (T_{air} - T_{p_2}) \quad (15)$$

The Stefan–Boltzmann constant $\sigma = 5.669 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$. The specific heat of plate 1 and plate 2 C_{p1} , C_{p2} such as for aluminum frame is 897 J/kg K. The specific heat $C_{p\text{air}}$ such as for air inside cylinder is 1005 J/kgK. The solar absorptance coefficients for surface materials plates α_{p1} , α_{p2} are 0.95 and 0.25. The density for aluminum ρ_{p1} and ρ_{p2} is 2712 kg/m³ and for air ρ_{air} are 1.205 kg/m³. The thickness of each layer, $d_{p1, p2, \text{air}}$ are 0.001, 0.001, and 0.015 m.

Convective heat transfer is one of the major modes of heat transfer and convection is also a major mode of mass transfer in fluids. The general equations for thermal coefficients as connective heat transfer, Nusselt number, Grasshof number, Prandtl number coefficients are shown in Eqs. (16)–(19) [24].

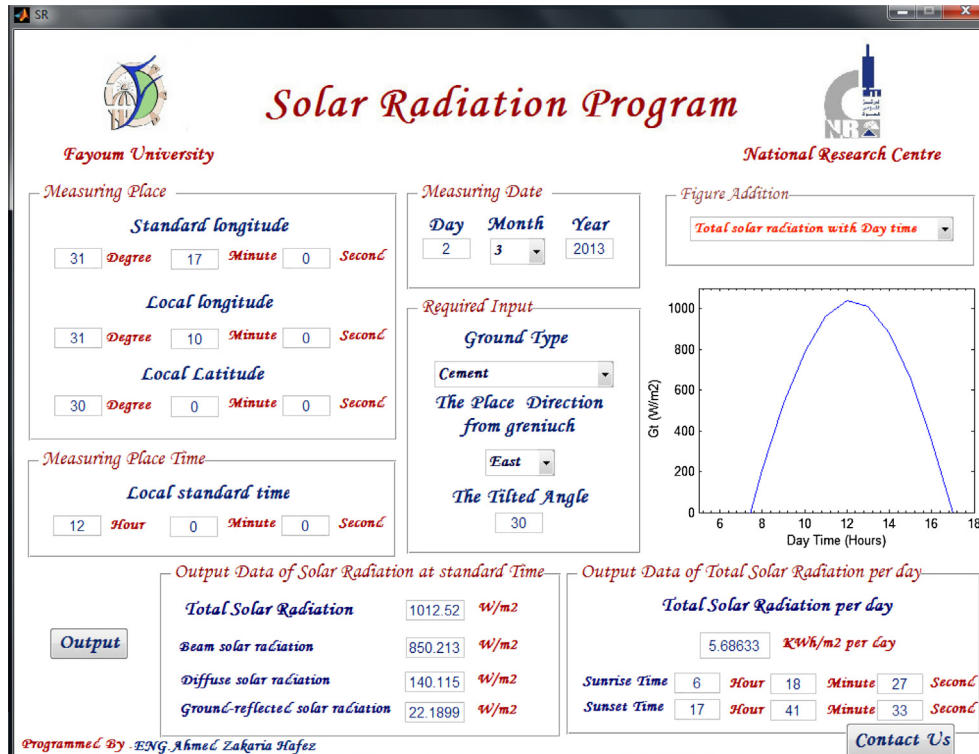
$$h_c = \frac{K}{L} N_u \quad (16)$$

$$N_u = a(G_r \times P_r)^n \quad (17)$$

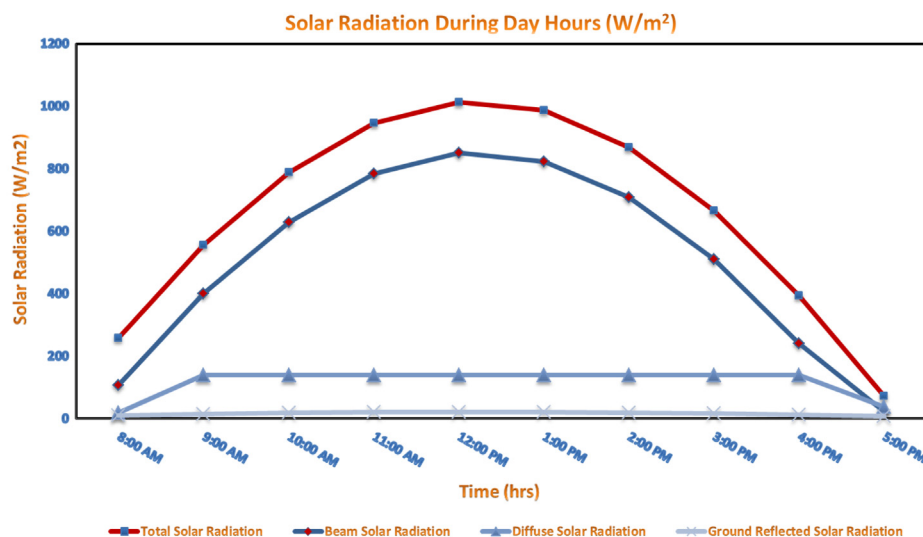
$$G_r = \frac{g \beta \rho^2 L^3 (T_1 - T_2)}{\mu^2} \quad (18)$$

$$P_r = \frac{C_p \mu}{K} \quad (19)$$

The general equation for Nusselt number equation at natural convection is shown in Eq (17). Convection heat transfer values are very specific to the geometry of the surface and the heat



(a)



(b)

Fig. 9. Estimation of daily solar radiation on an inclined plane (south facing – Tilted 30°) during day hours in Giza on 2 March 2013. (a) Matlab GUI/Interface simulation model (b) solar radiation data during day hours.

transfer conditions as shown in Table 2, we can estimate a, n constants values from Table 2 [24]:

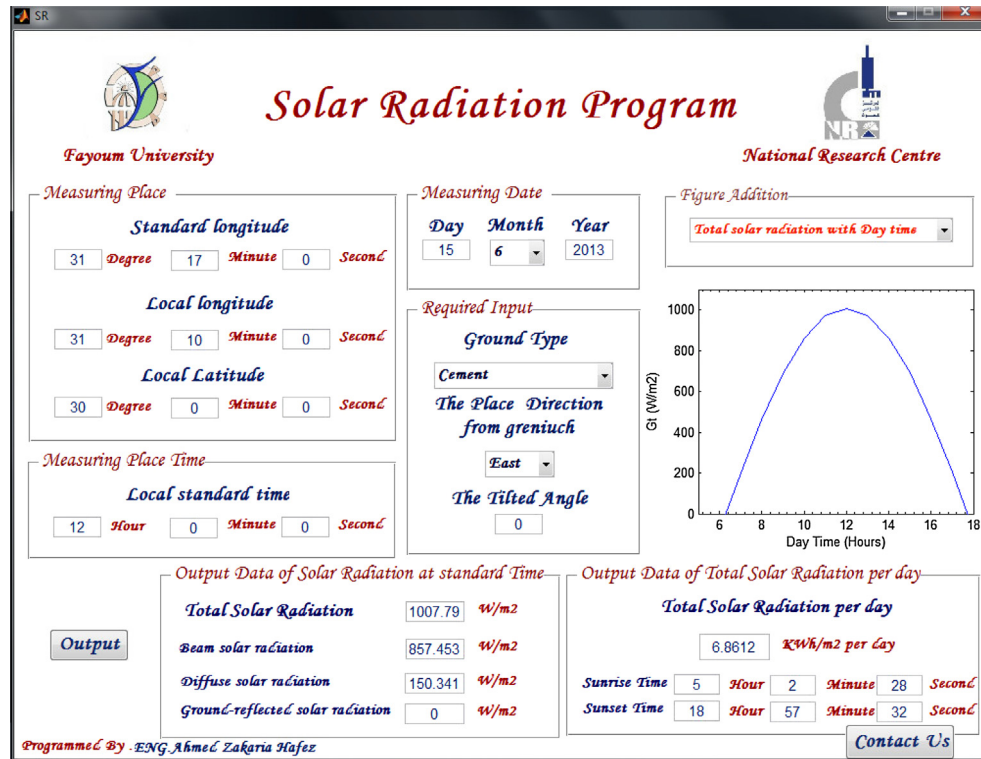
As shown in Fig. 2c for Stirling engine container plate 1, the outer radius of plate 1 is 5.5 cm, and the inner radius hole of plate 1 is 1 cm. The length (L) between plate 1 and plate 2 is 1.5 cm. The thickness of each layer, $d_{p1,p2,air}$ are 0.001, 0.001, and 0.015 m.

The specific heat $C_{p,air}$ such as for air inside cylinder is 1.005 kJ/kg K. The density for air ρ_{air} is 1.205 kg/m³. The solar absorptance coefficients for surface materials plates α_{p1}, α_{p2} are 0.95 and 0.25. The thermal Conductivity of air K_{air} is 0.0257 W/m K. The

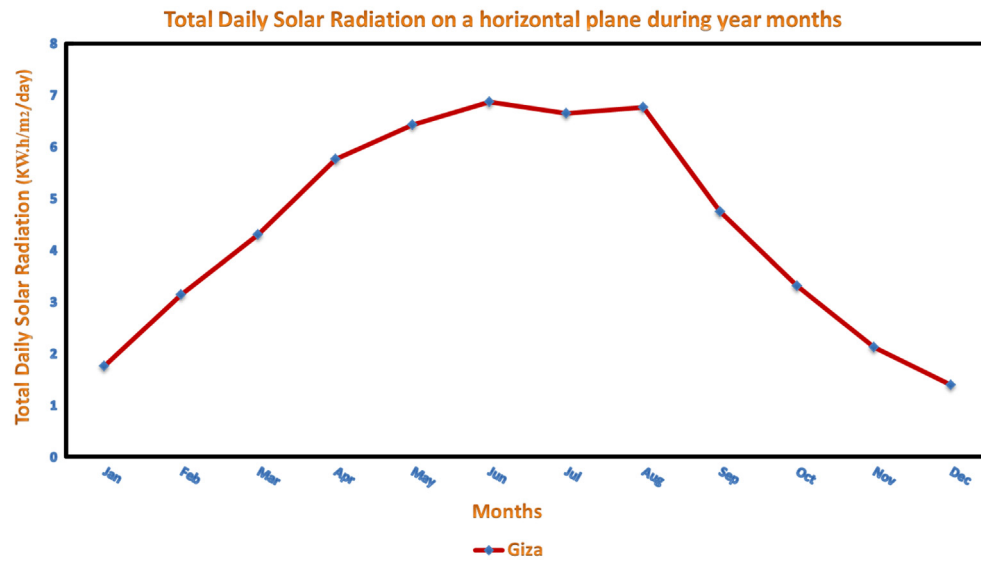
coefficient of thermal expansion of air β_{air} is 3.43 1/K. The dynamic viscosity of air μ_{air} is 1.983×10^{-5} g/ms. The gravity of acceleration g is 9.8 m/s².

Eq. (16) is a general relationship for connective heat transfer coefficient. As shown in Eq. (20), it can be reduced to identified forms for four layers of Stirling engine container

$$\begin{aligned} h_{c,a_1-p_1} &= 0.895055(T_{a_1} - T_{p_1})^{0.33}, h_{c,p_2-a_2} = 0.903129(T_{p_2} - T_{a_2})^{0.33}, \\ h_{c,p_1-air} &= 28.0346(T_{p_1} - T_{air})^{0.25}, h_{c,air-p_2} = 28.0346(T_{air} - T_{p_2})^{0.25} \end{aligned} \quad (20)$$



(a)



(b)

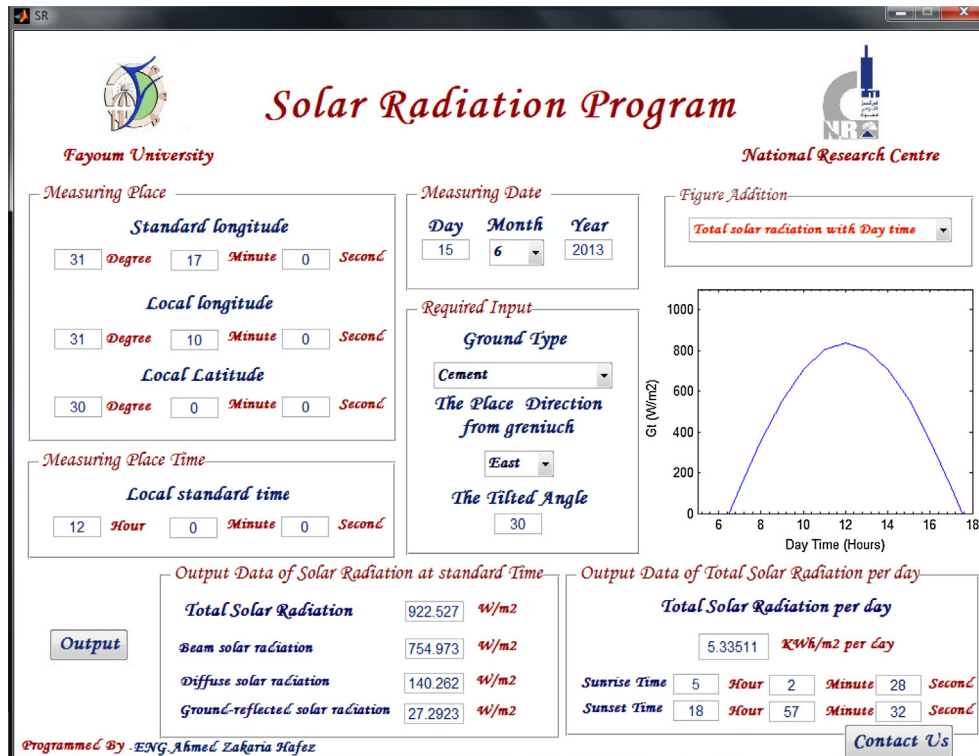
Fig. 10. Estimation of daily solar radiation on a horizontal plane during year months at Giza. (a) Matlab GUI/Interface simulation model. (b) Solar radiation data during year months.

Coefficient ε is the layer emissivity, and F is the view factor between two surfaces. The emissivity of the upper aluminum plate is 1. The emissivity of the lower plate for aluminum is 0.77. The solar absorptance coefficients for surface materials plates α_{p_1} , α_{p_2} are 0.95 and 0.25 [25].

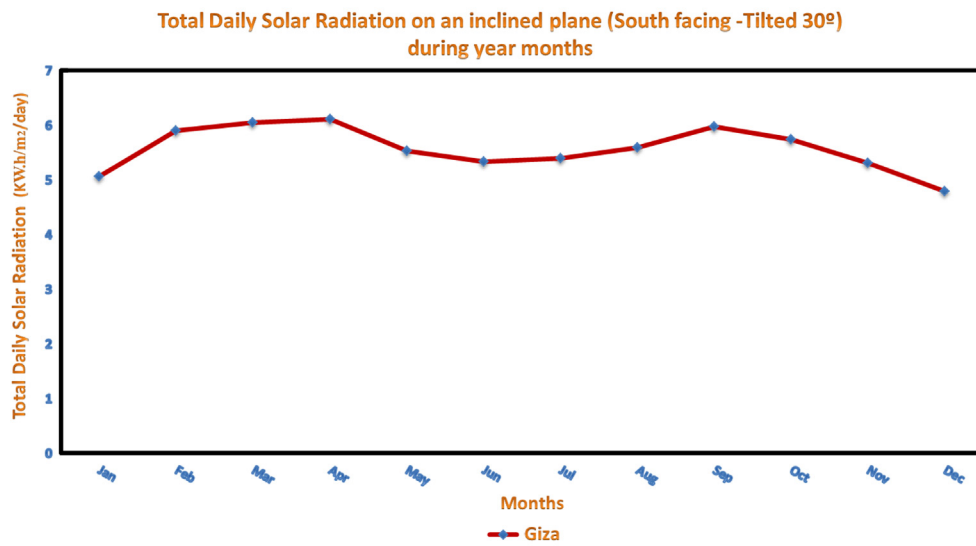
$$F_{p_1-p_2} = \frac{X-Y}{2} = 0.894, F_{a_1-p_1} = \frac{1+\cos(\beta)}{2} = 1, F_{p_2-a_2} = \frac{1+\cos(\beta)}{2} = 1 \quad (21)$$

The view factor from the inner side for aluminum upper plate to the inner side for lower plate $F_{p_1-p_2}$ is 0.894, and 1 from the outer surrounding air to the outer side for upper aluminum plate $F_{a_1-p_1}$. The view factor from the outer side for aluminum lower plate to the outer surrounding air $F_{p_2-a_2}$ is 1 as shown in Eq. (21).

As shown in Eqs. (13)–(15), the energy balance equations at plate 1 and plate 2 and at the container midpoint in the condition of steady state can be reduced too much simpler forms as Eqs. (22)–(24) as follow:



(a)



(b)

Fig. 11. Estimation of daily solar radiation on an inclined plane (South facing – Tilted 30°) during year months at Giza. (a) Matlab GUI/Interface simulation model. (b) Solar Radiation Data during year months.

$$0.95G_t + 0.895055(T_{a_1} - T_{p_1})^{1.33} + 5.669 \times 10^{-8}(T_{a_1}^4 - T_{p_2}^4) \\ = 28.0346(T_{p_1} - T_{air})^{1.25} + 5.068 \times 10^{-8}(T_{p_1}^4 - T_{p_2}^4) \quad (22)$$

$$28.0346(T_{air} - T_{p_2})^{1.25} + 4.9 \times 10^{-8}(T_{p_1}^4 - T_{p_2}^4) \\ = 0.903129(T_{p_2} - T_{a_2})^{1.33} + 4.36 \times 10^{-8}(T_{p_2}^4 - T_{a_2}^4) \quad (23)$$

$$T_{air} = \frac{T_{p_1} + T_{p_2}}{2} \quad (24)$$

5. Stirling engine power output modeling using Beale formula

In mechanical engineering, the Beale number is a parameter that characterizes the performance of Stirling engines. It is often used to estimate the power output of a Stirling engine design. For engines operating with temperature differential, typical values

for the Beale number range from (0.11) to (0.15); where a larger number indicates higher performance. To estimate the power output of an engine, nominal values are estimated for the Beale number (as shown in Fig. 7), pressure, swept volume and frequency, then the power is calculated as the product of these parameters, as follows:

$$P = B_n P_m V F \quad (25)$$

$$P_m V = m R T_{air} \quad (26)$$

$$P_m = \rho_{air} R T_{air} = 323.49 T_{air} \quad (27)$$

As shown in Fig. 2c for Stirling engine container plate 1, the outer radius (*r*) of plate 1 is 5.5 cm, and the length (*L*) between plate 1 and plate 2 is 1.5 cm.

$$V = \pi r^2 L = 142.6 \times 10^{-6} m^3 \quad (28)$$

$$P = B_n P_m V F = 0.046129674 B_n T_{air} F \quad (29)$$

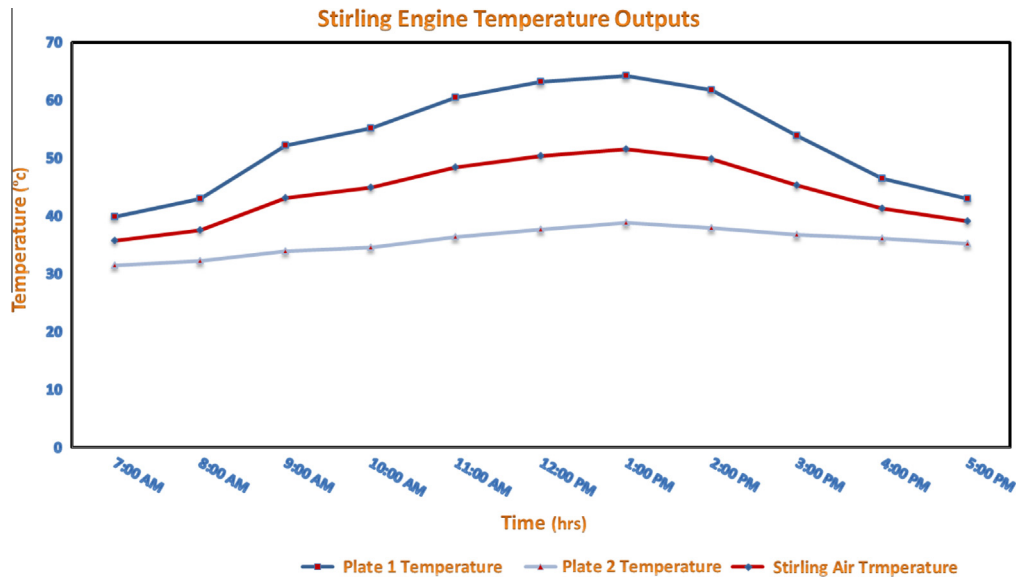


Fig. 12. Estimation of Stirling engine temperature for (Plate 1, Plate 2, and Air in Stirling engine container) during day hours on 2 March 2013 at Giza.

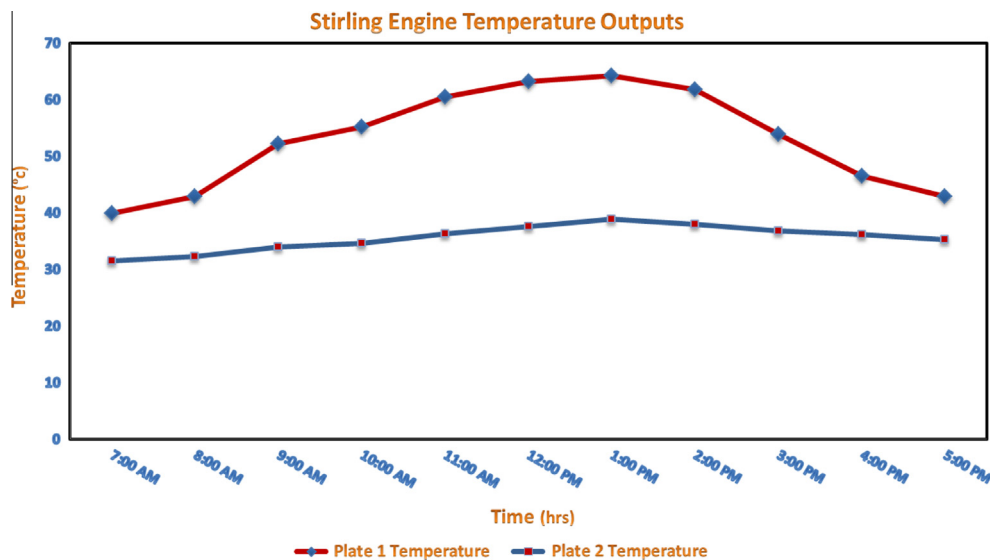


Fig. 13. Stirling engine temperature for (Plate 1 and Plate 2) in Stirling engine container during day hours on 2 March 2013 at Giza.

6. Results

Results from this study indicate that the engine performance increases with increasing solar intensity. The engine torque, shaft power, thermal efficiency, speed, and heating source temperature also increase with increasing solar intensity. In fact, it can be said that the maximum engine torque, shaft power, and thermal efficiency increase with increasing heating source temperature. The Beale number of this engine increases with decreasing temperature ratio or with increasing heating source temperature.

At the maximum solar intensity of 900 W/m^2 , or heat input of 8.55 J/s , the engine produced a maximum shaft power of 0.5171 W at 126 rpm , with a heating source temperature of 64.2°C and air temperature for heating container of 51.52°C .

Practical results and Simulation Matlab GUI/Interface model shows

- (1) Solar radiation data: solar radiation MATLAB®/GUI interfacing Results.

- (2) Thermal analysis in low temperature solar-powered Stirling engine for temperature of two plates of container and air in container:

- i. Solar powered Stirling engine temperature results.
- ii. Matlab simulation program for solar powered Stirling engine temperature results.

- (3) Power outputs data for Stirling engine:

- i. Solar powered Stirling engine power estimation results.
- ii. Matlab simulation program for solar powered Stirling engine power estimation results.

6.1. Solar radiation results

Fig. 8 shows the amount of daily solar insolation during the daytime. It is shown in Fig. 8 that the maximum solar radiation applied in Stirling engine is 900 W/m^2 at 12:00 PM. The results of daily solar radiation during daytime are showed.

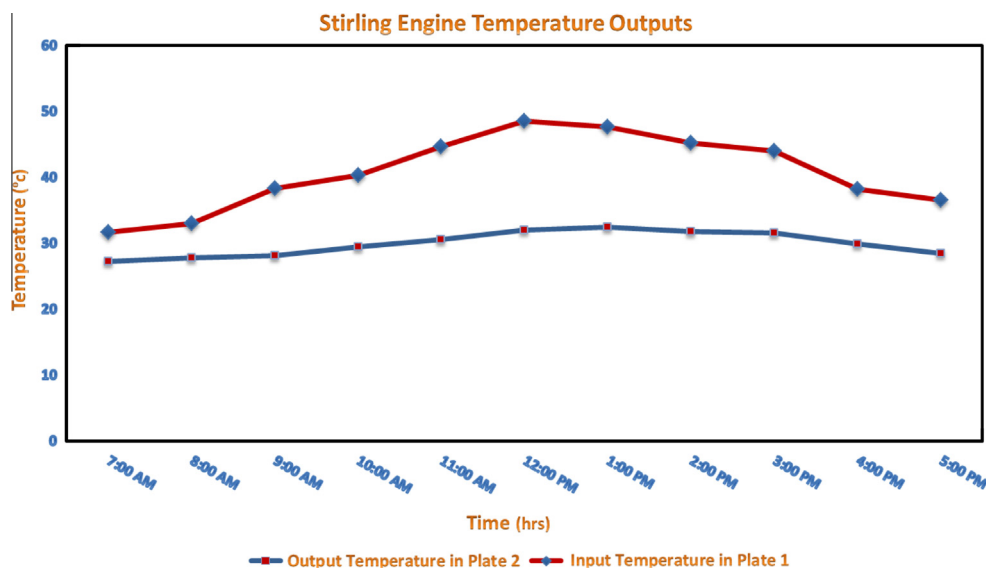


Fig. 14. Input and output temperature for (Plate 1 and Plate 2) in Stirling engine container during day hours on 2 March 2013 at Giza.

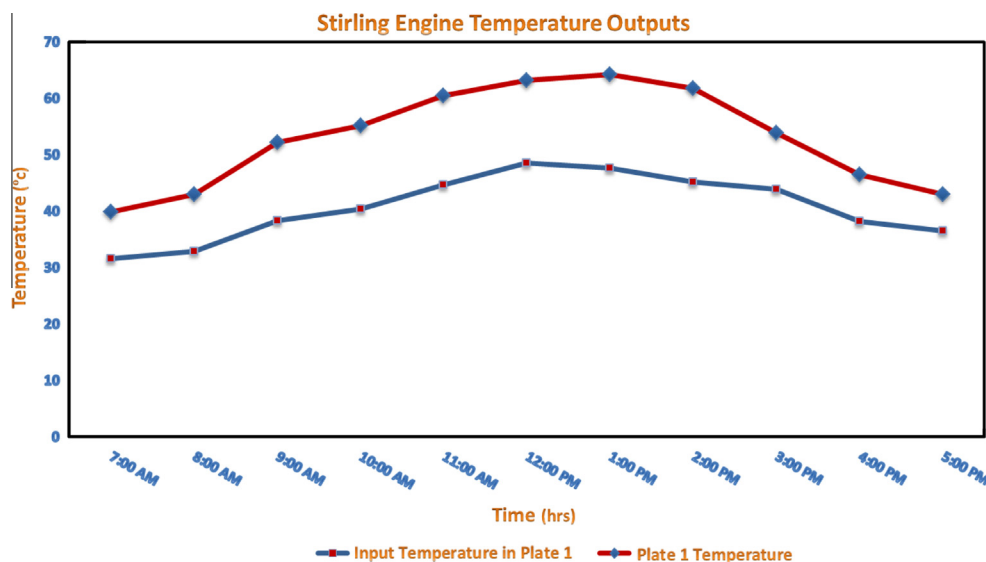


Fig. 15. Plate 1 and input temperature for Stirling engine container during day hours on 2 March 2013 at Giza.

Input data for place site (Giza) as latitude and longitude are showed in Table 3 [27] for Matlab simulation model. Fig. 9 shows Matlab/GUI estimation of Daily Solar Radiation on an inclined solar Stirling engine (South facing – Tilted 30°) during day hours on 2 March 2013 at Giza, the results in Fig. 10 shows estimation of daily solar radiation on a horizontal plane during year months at Giza, and the results in Fig. 11 shows estimation of daily solar radiation on an inclined plane (South facing – Tilted 30°) during year months at Giza.

6.2. Temperature results for Stirling engine

The temperature measurement experiments took place on 2 March 2013 from 07:00 AM to 5:00 PM by LM35 Sensor and data

acquisition (NI USB 6008). During these experiments the weather conditions were very good and were almost no clouds in the sky else from 4:00 PM to 5:00 PM. The experimental results of Stirling Engine Temperature for (Plate 1, Plate 2, and Air in Stirling engine container) are shown in Fig. 12. Fig. 12 shows the temperature variations of each layer (upper and lower plates) during the day-time. The maximum temperature of air inside Stirling engine was 51.52°C at 1:00 PM; the results in Fig. 12 clearly show the effects of the upper and lower plates temperature for air temperature inside heating container.

Fig. 13 show Stirling engine temperature for (Plate 1 and Plate 2) in Stirling engine container, Fig. 14 show input and output temperature for (Plate 1 and Plate 2), Fig. 15 show plate 1 and input temperature for Stirling engine container, and Fig. 16 show plate

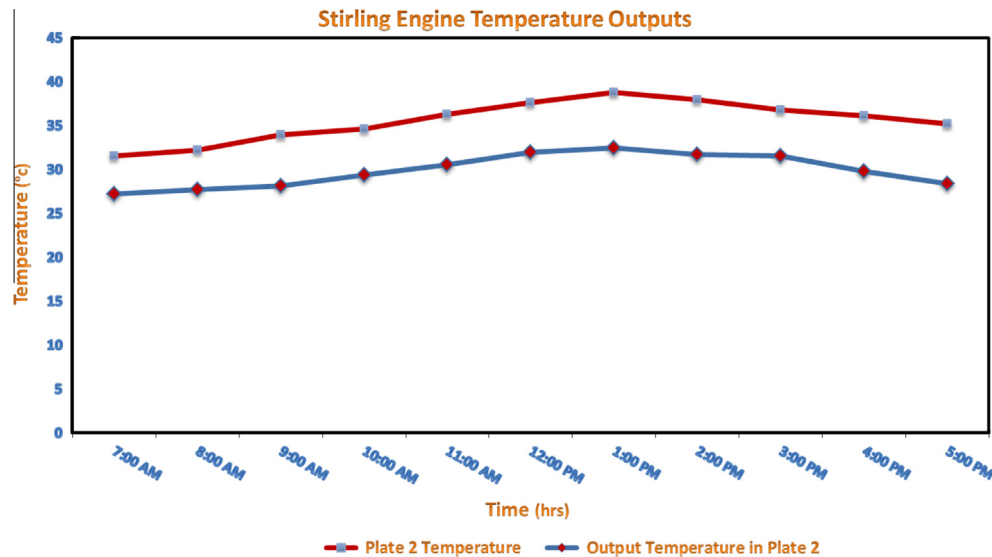


Fig. 16. Plate 2 and output temperature for Stirling engine container during day hours on 2 March 2013 at Giza.

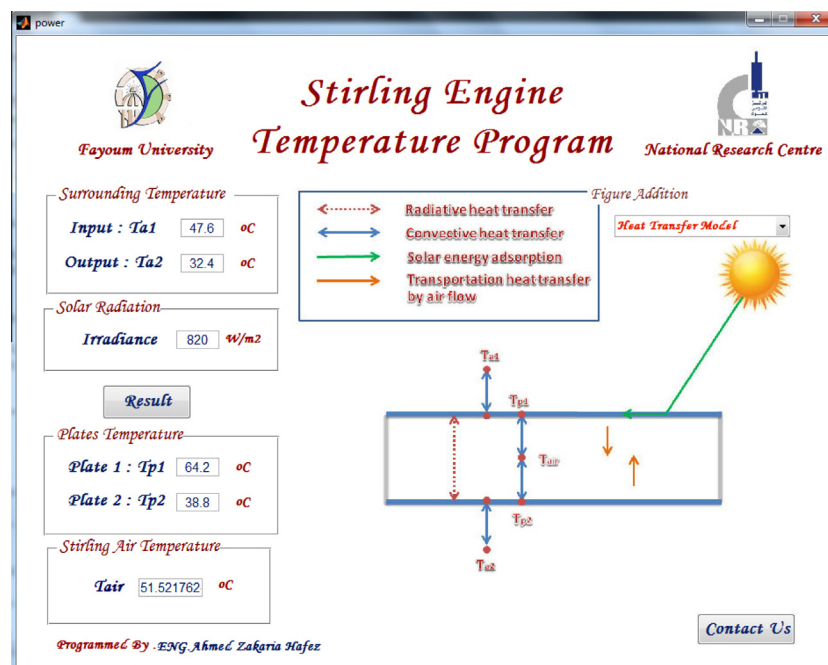


Fig. 17. Estimation of Stirling engine temperature by Matlab GUI/Interface simulation model on 2 March 2013 in Giza at 1:00 PM.

2 and output temperature for Stirling engine container during day hours on 2/3/2013 at Giza.

Simulation Matlab/GUI program shows in Fig. 17 estimation of solar Stirling engine temperature data during day hours on 2 March 2013 at Giza.

6.3. Power outputs results for Stirling engine

Figs. 18 and 19 show the results data of the solar-powered Stirling engine shaft power and rotating speed. Under the atmospheric pressure, the engine produces a maximum power of 0.5171 W at 126 rpm and air temperature for heating container 51.52 °C. The engine shaft power and rotating speed also increases with increasing solar intensity as shown in Figs. 18 and 19. The Beale number and the specific power demonstrated that the performance of this prototype, in terms of the power output, is on par with the high temperature design with more simple and less costly development.

Fig. 18 shows the Stirling engine speed during the daytime using Tachometer. It is shown in Fig. 18 that the maximum Stirling

engine speed applied in Stirling engine is 126 RPM at 1:00 PM. The results of power data for Stirling Engine during daytime are showed.

Fig. 19 shows the Stirling engine power estimation during the daytime using Beale formula. It is shown in Fig. 19 that the maximum Stirling engine power estimation by Stirling engine is 0.5171 watt at 1:00 PM.

Simulation Matlab GUI/Interface model showed the shaft power in Fig. 20 for the solar-powered engine at different solar radiation intensity.

Even though these figures are not so high, if we realize that the solar-powered Stirling engine is powered by a free heat source, this study would probably be more worthwhile as a starting point for the research of cheap solar-powered engines manufactured for rural and remote areas. The engine performance can be improved by increasing the precision of the engine parts and the heat source efficiency. The engine performance could be further increased if a better working fluid, e.g. helium or hydrogen, is used, instead of the air and/or by operating the engine at some degree of pressurization.

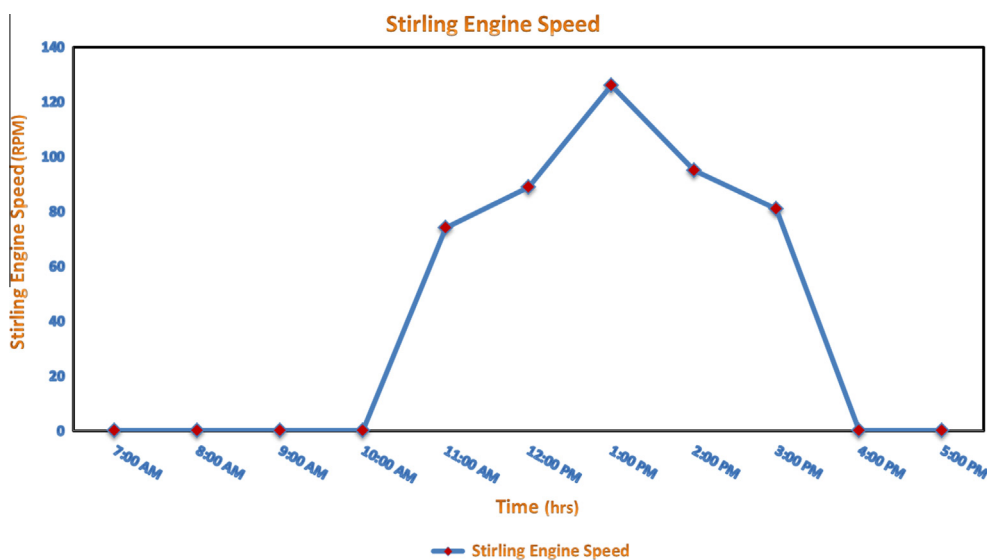


Fig. 18. Stirling engine speed during day hours on 2 March 2013.

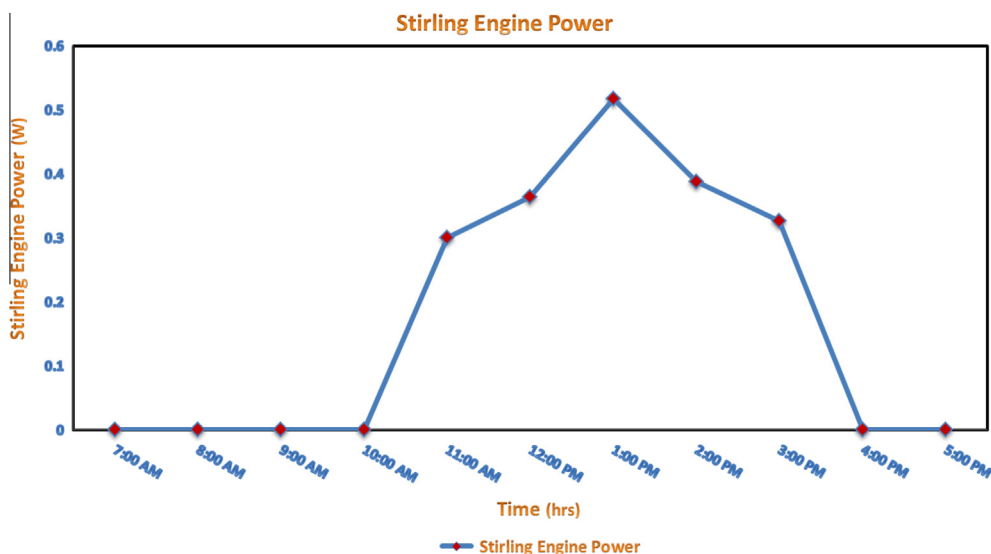


Fig. 19. Estimation of power output from Stirling engine during day hours in Giza on 2 March 2013.

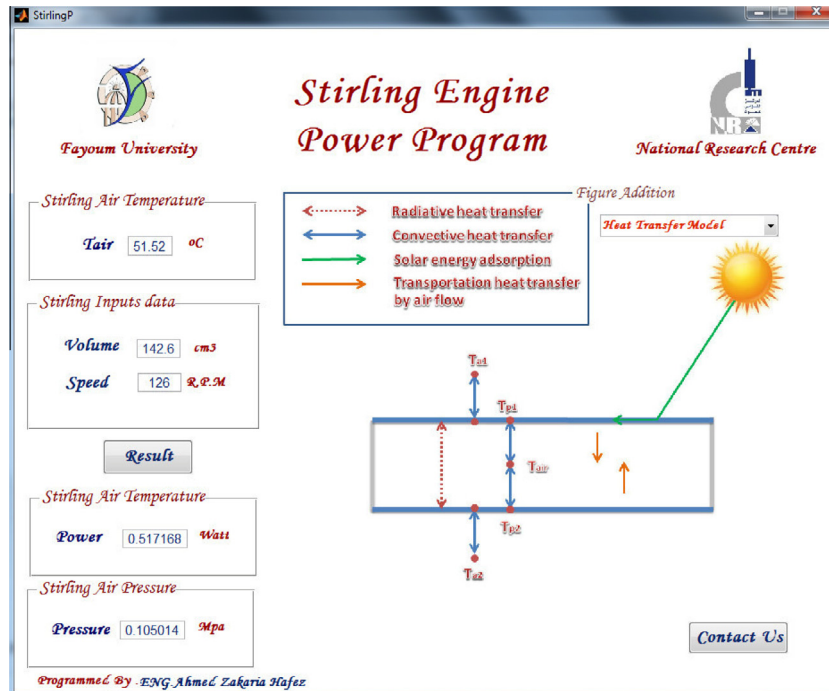


Fig. 20. Estimation of power output from Stirling engine by Matlab GUI/Interface simulation model in Giza at 1:00 PM.

7. Conclusion

This paper presents the modeling for a prototype of the solar-powered Stirling engine and mathematical model for the thermal analysis of the solar-powered low temperature Stirling engine with heat transfer is developed using Matlab program. The present analysis provides a theoretical guidance for designing and operating of the solar-powered low temperature Stirling engine system, as well as estimating output power from the solar Stirling engine using Matlab Program. Also, solar radiation simulation program presents the implementation of a solar radiation data using MATLAB/GUI interface. The model is developed using basic equations of the solar radiation including effects of time change and different location of sites.

Results from this study indicate that the engine performance increases with increasing solar intensity. The engine shaft power, speed, and heating source temperature also increase with increasing solar intensity. In fact, it can be said that the maximum engine shaft power increase with increasing heating source temperature. The Beale number of this engine increases with decreasing temperature ratio or with increasing heating source temperature. At the maximum solar intensity of 900 W/m^2 , or heat input of 8.55 J/s , the engine produced a maximum shaft power of 0.5171 W at 126 rpm , with a heating source temperature of 64.2°C and air temperature for heating container of 51.52°C .

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